

Why Teaching is Difficult

I've been giving this lecture to first-year classes for over twenty-five years. You'd think they would begin to understand it by now.

John Littlewood, Mathematician

“You Can’t Miss It”

When I first visited the US I was surprised to realize that whenever I asked for directions the person who gave the instructions ended with “You can’t miss it.” In most cases, I missed. If so, what do people mean by “you can’t miss it”? Only this: “I can’t understand the problem. As someone who is familiar with the way, I don’t see why it’s difficult.”

A person giving directions has lots of little pieces of knowledge to guide him: a bush at one point, a trash-can at another, a road sign at a third. He has never put words to all these, and therefore he does not convey them. Unheedingly, he assumes that they also exist in the mind of the receiver.

For those who know, it is difficult to understand what others do not understand. Therefore, one should always keep in mind the ancient Hebrew sages’ saying: “The shy cannot learn nor the impatient teach.” Both teacher and student must be aware of the fact that the teacher knows something the student does not, that it is in both of their interests to bridge the gap, and that they both need to make a common effort to do so.

Mathematical Subtleties

The bush, the trash-can and the road sign exist in mathematics as well. In mathematics, these are the small subtleties that are so easily missed. Here is an example of such a subtlety: the various meanings of subtraction.

During a first grade lesson I observed, the children looked at a drawing: three green apples and two red ones. They were supposed to tell “arithmetical stories” based on the drawing, one of addition and one of subtraction. (We will see the importance of such “stories”

when we discuss the meaning of arithmetical operations). They had no difficulty with the first task. “I had three green apples and two red apples. How many did I have altogether?” When they came to the subtraction story, confusion prevailed — and as usually happens in elementary school, it manifested as inattention. Eventually one of the children said: “I had five apples. I ate two. How many do I have left?”



It is easy to tell an addition story about this picture.
Can you also tell a subtraction story?

The problem was that this wasn't the “correct” story. It wasn't based on the drawing. The drawing doesn't show two apples disappearing, by being eaten or in any other way. That is why the children found the task difficult.

The difficulty is derived from a subtlety: Subtraction has more than one meaning. There is the meaning of “take-away,” where objects are removed: I had 5 balloons, 2 of them popped, how many do I have left? This is the meaning the child used in his story — his apples disappeared. But there is also the meaning of “whole-part,” where nothing disappears. There are 5 children in a group and 2 of them are boys. How many girls are there? Here, too, the exercise is $5 - 2$, but the meaning is different. This is the meaning depicted in the drawing. The story that fits the drawing is: “I have 5 apples, 2 of them are red. How many of them are green?” To avoid confusion and anxiety, this should be clarified for the children.

Children Think Differently

An additional obstacle to recognizing missing layers is that it is difficult for adults to grasp a simple truth: Children think differently. As the poet Theodore Roethke is quoted in Introduction, “A poet must have his childhood close at hand.” A poet must be able to connect mainly with his feelings. A good teacher needs to connect with the way the child within her thinks.

It is not easy at all. Patterns of thought change with age, and as we grow older we tend to forget them, in mathematics as in all other areas of life.